Mathematics: Subject-specific guidance

See also: EE generic guide and EE Teacher support material

For a general introduction to undertaking an EE in mathematics, see Mathematics: An introduction.

Overview

An extended essay (EE) in mathematics is intended for students who are writing on any topic that has a mathematical focus and it need not be confined to the theory of mathematics itself.

Essays in this group are divided into six categories:

- the applicability of mathematics to solve both real and abstract problems
- the beauty of mathematics—eg geometry or fractal theory
- the elegance of mathematics in the proving of theorems—eg number theory
- the history of mathematics: the origin and subsequent development of a branch of mathematics over a period of time, measured in tens, hundreds or thousands of years
- the effect of technology on mathematics:
- in forging links between different branches of mathematics,
- or in bringing about a new branch of mathematics, or causing a particular branch to flourish.

These are just some of the many different ways that mathematics can be enjoyable or useful, or, as in many cases, both.

Choice of topic

The EE may be written on any topic that has a mathematical focus and it need not be confined to the theory of mathematics itself.

Students may choose mathematical topics from fields such as engineering, the sciences or the social sciences, as well as from mathematics itself.

Statistical analyses of experimental results taken from other subject areas are also acceptable, provided that they focus on the modelling process and discuss the limitations of the results; such essays should not include extensive non-mathematical detail.

A topic selected from the history of mathematics may also be appropriate, provided that a clear line of mathematical development is demonstrated. Concentration on the lives of, or personal rivalries between, mathematicians would be irrelevant and would not score highly on the assessment criteria.

It should be noted that the assessment criteria give credit for the nature of the investigation and for the extent that reasoned arguments are applied to an appropriate research question.

Students should avoid choosing a topic that gives rise to a trivial research question or one that is not sufficiently focused to allow appropriate treatment within the requirements of the EE.

Students will normally be expected either to extend their knowledge beyond that encountered in the Diploma Programme mathematics course they are studying or to apply techniques used in their mathematics course to modelling in an appropriately chosen topic.

However, it is very important to remember that it is an essay that is being written, not a research paper for a journal of advanced mathematics, and no result, however impressive, should be quoted without evidence of the student's real understanding of it.

Examples of topics

These examples are just for guidance. Students must ensure their choice of topic is focused (left-hand column) rather than broad (right-hand column).

Focused topics	Broad topics
Prime numbers in cryptography	Prime numbers
The Hausdorff dimension of fractal sets	Fractals
Continued fractions in birth–death processes	Continued fractions
The proof of the law of quadratic reciprocity	CF Gauss: the mathematician
Using graph theory to minimize cost	Graph theory

Treatment of the topic

Whatever the title of the EE, students must apply good mathematical practice that is relevant to the chosen topic, including:

- data analysed using appropriate techniques
- arguments correctly reasoned
- situations modelled using correct methodology
- problems clearly stated and techniques at the correct level of sophistication applied to their solution.

Research methods

Students must be advised that mathematical research is a long-term and open-ended exploration of a set of related mathematical problems that are based on personal observations. The answers to these problems connect to and build upon each other over time.

Students' research should be guided by analysis of primary and secondary sources.

A primary source for research in mathematics involves:

- data-gathering
- visualization
- abstraction
- conjecturing
- proof.

A secondary source of research refers to a comprehensive review of scholarly work, including books, journal articles or essays in an edited collection.

A literature review for mathematics might not be as extensive as in other subjects, but students are expected to demonstrate their knowledge and understanding of the mathematics they are using in the context of the broader discipline, for example how the mathematics they are using has been applied before, or in a different area to the one they are investigating.

Writing the essay

Throughout the EE students should communicate mathematically:

- describing their way of thinking
- writing definitions and conjectures
- using symbols, theorems, graphs and diagrams
- justifying their conclusions.

There must be sufficient explanation and commentary throughout the essay to ensure that the reader does not lose sight of its purpose in a mass of mathematical symbols, formulas and analysis.

The unique disciplines of mathematics must be respected throughout. Relevant graphs and diagrams are often important and should be incorporated in the body of the essay, not relegated to an appendix. However, lengthy printouts, tables of results and computer programs should not be allowed to interrupt the development of the essay, and should appear separately as footnotes or in an appendix. Proofs of key results may be included, but proofs of standard results should be either omitted or, if they illustrate an important point, included in an appendix.

Examples of topics, research questions and suggested approaches

Once students have identified their topic and written their research question, they can decide how to research their answer. They may find it helpful to write a statement outlining their broad approach. These examples are for guidance only.

Topic	**The geometry of navigation**
Research question	What was the role of mathematics, and geometry in particular, in navigation when we relied on the stars? Does it still play a part now we have man-made satellites?
Approach	Using one of the two geometric representations of the Earth (spherical or ellipsoidal), describe how maps and charts were produced to assist navigators in the past.

Topic	**Square–triangular numbers and Pell's equation**
Research question	How many square numbers are also triangular numbers, where are they and what other problems lead to Pell's equation?
Approach	A description of square and triangular numbers, and how the locations of numbers that are both are solutions of Pell's equation. Some other problems, perhaps in number theory and geometry, that lead to the equation could be described, with a brief history of the equation included.

Topic	**The exponential function and the measurement of age and growth**
Research question	How does the exponential function, and its calculus, inform areas of science such as nuclear physics, geology, anthropology or demography?
Approach	Use one of the settings where exponential growth applies, perhaps modelling the world's population, to describe the phenomenon. Show how it is applicable in mathematical models of other real situations.

Topic	**Approximation of irrational numbers by rational numbers**
Research question	How well can π, e, $\sqrt{2}$ and other irrationals be approximated by rational numbers?
Approach	Use the decimal representation of irrational numbers as a starting point to introduce approximation by rationals. Show how a continued fraction expansion of an irrational can also provide rational approximation, and discuss error bounds and orders of approximation.

Topic	**Archimedes' calculation of areas**
Research question	What is the legacy of Archimedes' calculations of circular and parabolic areas in today's methods of integration?
Approach	Describe how Archimedes determined the area of a circle by using inscribed polygons, leading also to his measurement of π. Continue with a description of his method of discovery for calculating the area of a parabola.

An important note on "double-dipping"

Students must ensure that their EE does not duplicate other work they are submitting for the Diploma Programme. For example, students are not permitted to repeat any of the mathematics in their IA in their EE, or vice versa.

The mathematics EE and internal assessment

An EE in mathematics is not an extension of the internal assessment (IA) task. Students must ensure that they understand the differences between the two.

- The EE is a more substantial piece of work that requires formal research.
- The IA is an exploration of an idea in mathematics.

It is not appropriate for a student to choose the same topic for an EE as the IA. There would be too much danger of duplication and it must therefore be discouraged.

Supervisors play an important role in guiding students on these distinctions. Students risk their diploma if academic misconduct is detected.

Interpreting the EE assessment criteria

Criterion A: Focus and method

(Strands: Topic, Research question, Methodology)

In mathematics the title of the essay can by itself clearly describe the topic and/or aim of the essay. It must not be too long and any necessary clarification of it, together with a clear indication of the mathematical areas and the techniques, should be provided early in the essay.

For example, "Methods for approximating π throughout history". In this essay I will describe methods of approximating π from the work of Archimedes to the use of infinite series, infinite products and continued fractions in subsequent periods." In other words, the focus and purpose of the essay must be made clear to the reader and appropriately related to the knowledge and understanding in context. This is clearly demonstrated when the research question indicates the mathematical techniques to be applied.

The sources consulted must be sufficient and each must contribute to the research focus of the essay.

The essay must be set out in sequential form in the manner of good mathematical writing, that is each section following on from and connected to the previous one.

A sharply written clear focus and research question can help the student ensure the essay remains within 4,000 words.

Criterion B: Knowledge and understanding

(Strands: Context, Subject-specific terminology and concepts)

The essay must show clear evidence of understanding of the mathematics that is relevant to the focus of the essay. Students will not be rewarded for attempting to exhibit a wider knowledge of mathematics that is not essential to exploring the research question.

For example, in an essay on fractals, students must describe the mathematical concepts that underlie them without resorting to advanced theorems and results in analysis.

Students can demonstrate their understanding by:

- giving accurate and complete explanations of subject-specific terminology
- making knowledgeable comments on source material
- using source material in a relevant and appropriate way.

Students should ensure that the essay's content is accessible to readers with a strong interest in the subject as well as to those with an advanced knowledge of it.

Students need to clearly communicate and explain their mathematics. They must not just talk about it but actually do the mathematics, and must show all steps in mathematical reasoning to make it clear that they understand it.

Students must make sure definitions are fully explained. If a theorem is used whose proof is too difficult, it should at least be explained by a clear example. Throughout, students need to demonstrate that they fully understand what they are doing.

Criterion C: Critical thinking

(Strands: Research, Analysis and Discussion and evaluation)

Students should be aware of the particular demands of critical thinking in mathematics.

At each opportunity in the essay, students must demonstrate their abilities in:

- correct deductive reasoning and argument
- establishing hypotheses
- formulating mathematical models.

For example, in the use of statistics to establish a hypothesis, students must collect the correct data, then display summary data and graphs, so that they choose, apply and interpret correctly the appropriate test or tests.

Students' discussion and evaluation of their results should be concise.

It is important that students do the mathematics rather than merely describe it. They must show the steps in the algebra to demonstrate that they really understand what is going on. If they take any element from a source, they must cite that source.

Students should prove conjectures that can readily be proved. The essay must not just quote results; there must be evidence of the student doing mathematics.

Criterion D: Presentation

(Strands: Structure, Layout)

This criterion relates to the extent to which the essay conforms to accepted academic standards in relation to how research papers should be presented. It also relates to how well these elements support the reading, understanding and evaluation of the essay.

Students must provide a section and subsection structure to their essays, with appropriate informative headings. Students should aim to demonstrate their mastery of appropriate concepts and an ability to present these in an effective way using mathematical means. Concise, elegant mathematics supported by graphs, diagrams and important proofs that do not interrupt the development of the essay are encouraged.

Use of charts, images and tables

Diagrams and pictures should be in the text, immediately close to an explanation of them. Small data tables can be included in the body of the essay but larger ones should appear as an appendix, with means, standard deviations, correlation coefficients etc given in the text.

Students should include computer routines only if they are absolutely necessary for the understanding of the essay. These must always appear as an appendix.

Any material that is not original must be carefully acknowledged, with specific attention paid to the acknowledgment and referencing of quotes and ideas. This acknowledgment and referencing is applicable to audiovisual material, text, graphs and data published in print and electronic sources. If the referencing does not meet the minimum standard as indicated in the guide (name of author, date of publication, title of source and page numbers as applicable), and is not consistently applied, work will be considered as a case of possible academic misconduct.

A bibliography is essential and has to be presented in a standard format. Title page, table of contents, page numbers, etc must contribute to the quality of presentation.

Word count is rarely an important factor in a good mathematics EE. Since equations and formulas (indicating the student's mathematical reasoning) are not included in the word count, a substantial essay can be produced that contains comparatively few words.

Concise, elegant mathematics supported by graphs, diagrams and important proofs that do not interrupt the development of the essay are encouraged. However, an essay that is excessive in length will be penalized, especially if this is because of unnecessary content. Students should be aware that examiners will not read beyond the 4,000-word limit, or assess any material presented past this.

There is no mandatory minimum length for an essay in mathematics, and credit will be given for organizing the content in an efficient and readable style, rather than for a page or word count. Mastery of appropriate concepts, and an ability to present these in an effective way using mathematical means, should be the aim. Students should use an appendix as appropriate (eg for large amounts of raw data or for computer routines). However, any mathematics that is essential to the understanding of the essay must appear in the main body of the essay.

Criterion E: Engagement

(Strands: Reflections on planning and progress)

This criterion assesses the student's engagement with their research focus and the research process. It will be applied by the examiner at the end of the assessment of the essay, and is based solely on the candidate's reflections as detailed on the RPPF, with the supervisory comments and extended essay itself as context.

Students are expected to provide reflections on the decision-making and planning process undertaken in completing the essay. Students must demonstrate how they arrived at a topic as well as the methods and approach used. This criterion assesses the extent to which a student has evidenced the rationale for decisions made throughout the planning process and the skills and understandings developed.

For example, students may reflect on:

- the approach and strategies they chose, and their relative success

- the *Approaches to learning* skills they have developed and their effect on the student as a learner
- how their conceptual understandings have developed or changed as a result of their research
- challenges they faced in their research and how they overcame these
- questions that emerged as a result of their research
- what they would do differently if they were to undertake the research again.

Effective reflection highlights the journey the student has engaged in through the EE process. Students must show evidence of critical and reflective thinking that goes beyond simply describing the procedures that have been followed.

The reflections must provide the examiner with an insight into **student** thinking, creativity and originality within the research process. The **student** voice must be clearly present and demonstrate the learning that has taken place.